7502/MP1 **OCTOBER 2008**

Paper I — MATHEMATICAL PHYSICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) (i) State and prove Stokes theorem.

(ii) Verify Stokes theorem for the vector $F = (z, x, y)$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy plane.

Or

(b) Define contravariant and co-variant tensors? And show that the law of transformation for a contravariant tensors is transitive.

2. (a) (i) Deduce the value of $\Gamma(1/2)$.

(ii) Find the relation between Beta and Gamma function.

(iii) Show that $\beta(m, n) = \beta(n, m)$.

Or

(b) Distinguish Dirac delta function from kronecker delta function and show that $\delta(x^2 - a^2) = 1/2a\,\{\delta(x + a) + \delta(x - a)\}$ where $a > 0$.

3. (a) Derive the Cauchy-Riemann's equation $f(z)$ is expressed in polar coordinates.

Or

(b) Find the cosine transformation of $X^n e^{-ax}$.

4. (a) (i) Starting from the definition of J_n(
prove that $J_{n-1}(x) + J_{n+1}(x) = 2n/x\, J_n(x)$.

(ii) Obtain the series solution of th
Hermite differential equation $y'' - 2xy' + 2ny = 0$ whe
$n = 2$.

Or

(b) (i) Prove that following recurren
relation for the Laguerre polynomi
$L_n(x) - nL'_{n-1} + nL_{n-1}(x) = 0$.

(ii) Construct the polynomial solution f
Legenders differential equation for $m = 0$.

5. (a) Derive the wave equation for a perfe
flexible stretched string and then construct a Fouri
series solution for it.

Or

(b) Construct the Greens function for the no
homogeneous problem $d^2u/dx^2 = f(x)$ with th
boundary conditions $u(0) = u(1) = 0$.