7492/KA1 **OCTOBER 2008**

Paper I — ALGEBRA — I

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

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PART A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. If G is a finite group, prove that the number of elements conjugate to a in G is the index of the normalizer of a in G.

2. If $O(G) = p^2$ where p is a prime number then prove that G is abelian.

3. Prove that a finite integral domain is a field.

4. Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Then prove that R is a field.

5. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

6. State and prove Gauss' lemma.

7. If $f(x)$ and $g(x)$ are primitive polynomials then prove that $f(x)g(x)$ is a primitive polynomial.

8. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

PART B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. State and prove Sylow theorem (first part).

10. Prove that $J[i]$ is a Euclidean ring.

11. Prove that every integral domain can be imbedded in a field.

12. If L is a finite extension of K and if K is a finite extension of F then prove that L is a finite extension of F.

13. If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exists an element $C \in F(a, b)$ such that $F(a, b) = F(C)$.

14. If p is a prime number and $p/O(G)$ then prove that G has an element of order p.
