

Paper I — ALGEBRA — I

---

(For those who joined in July 2003 and after)

Time : Three hours

Maximum : 100 marks

PART A — ( $4 \times 10 = 40$  marks)

Answer any FOUR questions.

Each question carries 10 marks.

1. State and prove Lagrange's theorem.
2. State and prove Cauchy's theorem for Abelian groups.
3. Prove that the homomorphic image of a solvable group is solvable.
4. Define maximal ideal. Let  $R$  be a commutative ring with unity. Prove that an ideal  $M$  of  $R$  is maximal ideal of  $R$  if and only if  $R/M$  is a field.
5. Let  $\alpha \in R$  be algebraic over  $F$  and  $f(x)$  the minimum polynomial of  $\alpha$  with  $\deg f(x) = n$ . Then prove that  $F(\alpha)$  has dimension  $n$  as a vector space over  $F$ .

