8728/M11 **APRIL 2009**

Paper I — L_p SPACES AND BANACH ALGEBRAS

(Held in November 2009)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

1. (a) Prove that a real differentiable function ϕ is convex in (a, b) if and only if $a < s < t < b \Rightarrow \phi'(s) < \phi'(t)$.

   (b) If ϕ is convex on (a, b) prove that ϕ is continuous on (a, b).

2. Let p and q be conjugate exponents, $1 < p < \infty$. Let X be a measure space with measure μ. Let f and g be measurable functions on X with range in $[0, \infty)$. Prove that

   (a) $\int_X fg\,d\mu \le \left[\int_X f^p d\mu\right]^{1/p} \left[\int_X g^q d\mu\right]^{1/q}$

   (b) $\left[\int_X (f+g)^p d\mu\right]^{1/p} \le \left[\int_X f^p d\mu\right]^{1/p} + \left[\int_X g^q d\mu\right]^{1/q}$