18. Find the centre and radius of the circle $x^2 + y^2 + z^2 = 225$, $2x - 2y + z = 27$.

19. Find the equation of the right circular cylinder of radius 2 whose axis passes through $(1, 2, 3)$ and has direction cosines proportional to $(2, -3, 6)$.

20. (a) If the second order partial derivatives of $\bar{f}$ be continuous, then prove that, $div(curl\ \bar{f}) = 0$.

(b) Prove that

$div(\phi\bar{u}) = (grad\ \phi).\ \bar{u} + \phi\ div\ \bar{u}$.

21. Evaluate the surface integral $\iint_S [yz\bar{i} + zx\bar{j} + xy\ \bar{k}]\ ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

22. Verify Green's theorem in the plane for $\vec{F} = (xy + y^2)\ \vec{i} + x^2\ \vec{j}$ over the closed curve bounded by $y = x$ and $y = x^2$.

Paper II — TRIGONOMETRY, ANALYTICAL GEOMETRY OF 3 DIMENSIONS AND VECTOR CALCULUS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — ($8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. Calculate $(1 + i)^{10}$.

2. Prove that $\dfrac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.

3. If $\tan(x + iy) = u + iv$, prove that $\dfrac{u}{v} = \dfrac{\sin 2x}{\sinh 2y}$.

4. Find the equation of the plane through the point $(1, -2, 3)$ and the intersection of the planes $2x - y + 4z = 7$ and $x + 2y - 3z + 8 = 0$.